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Effective Lagrangians for Scalar Fields and Finite Size Effects in Field Theory

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Abstract

We discuss the approach of effective field theory on a d-dimensional Euclidean space in a scalar theory with two different mass scales in the presence of flat surfaces. Then considering Dirichlet and Neumann boundary conditions, we implement the renormalization program in the $\lambda\varphi^4$ theory in a region bounded by two parallel hyperplanes in the one-loop approximation.

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1 Introduction

The quantum field theory of self-interacting scalar fields has long served as a laboratory for developing methods of analysis that can be applied to theories of more direct physical interest. Our purpose is to investigate the Euclidean field theory of two interacting scalar fields where after the construction of an effective theory for the light modes, in some limit, there is a decoupling between the light and heavy modes as stated by the Appelquist-Carrazzone theorem [1]. Aditionally we impose boundary conditions on the resulting theory in order to study finite-size effects and the renormalization program in systems where the translational symmetry is broken. The interest of the literature in the study of quantum fields in the presence of boundaries appears after the problem investigated by Casimir more than fifty years ago [2]. A complete review of this effect can be found in refs. [3] [4] [5].

In 1948, Casimir showed that neutral perfectly conducting parallel plates in vacuum attract each other, the effect can be interpreted as follows: the distortion of the vacuum fluctuations of the quantized electromagnetic field due to the presence of metallic plates renders the zero-point energy of the field into a measurable quantity. In the absence of any classical background, the renormalized vacuum expectation value of the Hamiltonian operator can be correctly defined by the Wick-ordered product. The main support for this procedure comes from the proof that for a relativistic field theory, the vacuum expectation value of the stress-energy tensor should vanish in order to ensure that the realization of the Poincaré generators in terms of the fields of the theory satisfy the correct commutation relations [6]. On the other hand, where external fields or macroscopic structures are present, more elaborated methods must be used to find the renormalized vacuum energy of the quantized field while avoiding undesirable divergences. A procedure that identifies the divergent contributions to the vacuum energy, such as a cut-off or any analytic regularization method, followed by a renormalization is mandatory. The fundamental idea of the Casimir renormalization procedure is that although formally divergent, the difference between the zero-point energies of two different physical configurations can be shown to be finite.

In quantum electrodynamics there is a standard argument used to support the implementation of a regularization procedure followed by renormalization to obtain the renormalized vacuum expectation value of the stress-energy tensor associated with the Maxwell field in the presence of boundaries. At high frequencies no real material is a perfect conductor. A wavelength cut-off corresponding to the finite plasma frequency must be included in the model. High energy modes are insensitive to the boundaries and only the low energy modes are affected by them. Consequently, in the study of quantum electrodynamics in the presence of conducting boundaries, in the generating functional for the n-point correlation functions this latter condition means that one may integrate out all the Fourier modes associated with the Dirac field and obtain an effective theory for the Maxwell field.

We would like to implement the above discussed situation in a very simple model of two Euclidean self-interacting scalar fields. We will analyse a theory with two massive scalar fields with different mass scales and we will be interested in obtaining the effective action for the light field. One could theoretically envision a theory with two massive fields with different mass scales on which the mass of the heavy field is smaller than the natural cut-off of the boundary, nevertheless in this paper we are not interested to discuss this situation. It is important to keep in mind that in order to construct an effective action that gives a correct description of the physics of the light modes in the presence of the boundaries, the Fourier modes associated with both fields with wavelength smaller than Λ^{-1} for some cut-off Λ must be integrated out, restricting the space of functions that we are integrating over (note that we are always assuming a sharp cut-off). Since dealing with functional integrals with cut-offs in general models is quite complicated, we will limit ourselves to a heavy field with Gaussian functional integrals. We are studying finite size effects for the light modes in two steps. In the first, we integrate over the modes of the heavy field, obtaining an effective action for the light field. In the second step we are taking the limit in which the decoupling theorem is valid ($m_2 \to \infty$), to regard the effective action as the fundamental action and we are assuming boundary conditions over the remaining light field. A combination of different analytic regularization procedures and a renormalization are able to eliminate the the usual bulk and also the additional surface divergences that appear in the theory. The final result of our procedure is that we have the effect of the compactification of one dimension, breaking the full translational invariance of the original theory. In this situation we have an effective field theory of the light modes, with finite size effects. Of course, the region outside the boundaries is the union of two simple connected domains and the renormalization of the interacting field theory in such region must be carried out along the same lines that we have to use in the interior region. For simplicity we are only focussing our analysis to the interior region.

There are many papers in the literature discussing quantum field theory in the presence of boundaries or macroscopic structures. The calculation of the radiative corrections to the renormalized energy density associated with the Maxwell field, in the presence of perfectly conducting plates, assuming no boundary conditions for the Dirac field was performed by Bordag et al [7]. Temperature corrections for this model were analyzed by Robaschik et al [8]. Bordag et al also studied the leading radiative correction to the renormalized energy, assuming that the paralel plates are represented by delta function potentials [9]. Using the approach of effective field theory, the radiative correction to the Casimir effect integrating out the fermionic degrees of freedom was examined by Kong and Ravndal and Ravndal and Thomassen [10]. A different approach was used by Falomir et al. [11]. These authors studying scalar fields in the presence of a spherical shell used a sharp cut-off, assuming that the boundary is transparent for the heavy modes while for the soft modes they assumed Dirichlet boundary conditions. Actor [12] studied two interacting scalar fields in the presence of macroscopic boundaries assuming that only one of the fields satisfies classical boundary conditions. Using the generalized zeta function method [13] the one-loop effective action was presented. More recently, Melnikov [14] investigated the low-energy effective action in a model with two scalar fields and also in quantum electrodynamics. Our treatment is very similar to the treatment developed by Melnikov in the study of the low-energy effective action for a theory with two interacting scalar fields.

It is important to point out that the combination of effective field theory and finite size effects can produce unexpected new phenomena. A well-known example of this situation is the

Scharnhorst effect [15]. Studying quantum electrodynamics between perfectly conducting plates, Scharnhorst concluded that the speed of light normal to the plates, exceeds light speed on vacuum, while parallel to the plates light travels with its vacuum speed, i.e., with that of light propagating in unbounded space. Further calculations by Barton [16] and also by Barton and Scharnhorst [17] confirmed the original result. The Scharnhorst effect is probably a consequence of the combined use of the effective Lagrangian in quantum electrodynamics and the presence of the plates, i.e., the Euler-Heisenberg Lagrangian density [18], derived many years ago and also rederived later by Schwinger [19]. As we already pointed out, the central issue of this effective Lagrangian is a derivative expansion of the photon effective action obtained by integrating out the fermionic field in the Maxwell-Dirac action. For an interesting discussion concerning the velocity of propagation of signals in different field theory models, see for example, [20].

Finite size effects that do not break translational invariance in quantum field theory also have been extensively studied in the literature [21] [22]. For translationally invariant systems, we can change from coordinate space to momentum space representation, the latter being a more convenient framework to analyze the divergences of the n-point Schwinger functions on which translational invariance is realized through conservation conditions. For systems where the translational invariance has been partially broken (so there is still translational invariance along certain directions) a more convenient representation for the n-point Schwinger functions is a mixed momentum-coordinate representation. Important references discussing the renormalization program in the presence of boundaries are the Symanzik [23] and also Diehl and Dietrich [24] papers.

In this paper we are studying the renormalization program in the presence of surfaces where a scalar field satisfies boundary conditions. We are interested in investigating a very simple model where we can construct an effective field theory for the light field on which the decoupling theorem holds. Further we consider an interacting Euclidean field theory of the light field in the presence of boundaries. We will consider a Casimir-like configuration where one of the coordinates, z, lies in the interval $[0, L]$ imposing Dirichlet-Dirichlet boundary conditions and for the sake of completeness we will also enquire about Neumann-Neumann boundary conditions.

The organization of the paper is as follows: In section II we introduce a simple model of two Euclidean interacting scalar fields. By integrating out the heavy modes associated with one of the fields, we are able to build the effective action for the remaining field. In section III we discuss a scalar field theory with boundary conditions due to the finite size of one of the coordinates and build the free two-point and four-point functions, both for Dirichlet-Dirichlet and Neumann-Neumann boundary conditions. In section IV we discuss the surface divergences of the one-loop two-point and also four-point function function. In section V we discuss the global approach, used to define the Casimir energy associated with a field in the presence of boundaries with well defined geometric shape.. Finally, section VI contains our conclusions. Throughout this paper we use $\hbar = c = 1$.

2 The Euclidean functional integral and the effective action

The goal of this section is to present a very simple model of two Euclidean self-interacting scalar fields where after the construction of an effective action [25] [26] [27] [28], and further imposition of the infinite mass limit for the heavy field, the decoupling theorem holds. For this purpose we start from a model with two different mass scales. We consider two real interacting massive scalar fields $\varphi_1(x)$ and $\varphi_2(x)$ with masses m_1 and m_2, respectively and regard the field $\varphi_2(x)$ as the heavy field ($m_2 >> m_1$). For this theory, we will rederive the Euclidean version for the Appelquist-Carrazonne theorem stating that for certain choice for the self-interacting $\varphi_1(x)$ part and in the limit $m_2 \to \infty$, there is a decoupling in the effective theory. The only effects of the heavy field $\varphi_2(x)$ being a modification of the value of the renormalized mass and the coupling constant of the light field $\varphi_1(x)$.

We start from the generating functional for the n-point Schwinger functions associated with two massive real fields in a d-dimensional Euclidean space given by

$$Z[j_1, j_2] = \mathcal{N} \int [d\varphi_1][d\varphi_2]\, e^{-S[\varphi_1,\varphi_2]+(source\, terms)}, \tag{1}$$

where $[d\varphi_1][d\varphi_2] = \prod_{x\in R^d} d\varphi_1(x) d\varphi_2(x)$ is an appropriate measure, $S[\varphi_1, \varphi_2]$ is the classical action associated with the Euclidean fields, and in the generating functional, $\mathcal{N}$ is a normalization. As usual, the n-point Schwinger functions of the theory can be obtained by functional differentiation with respect to the external sources $j_1(x)$ and $j_2(x)$. Since in this section our interest is to construct the effective theory for the light field, the introduction of the external sources in the functional integral is not important for our discussion. We consider the theory described by the following Euclidean Lagrangian density with two real scalar fields

$$\mathcal{L}(\varphi_1, \varphi_2) = \mathcal{L}_0(\varphi_1, \varphi_2) + \mathcal{L}_{int}(\varphi_1, \varphi_2), \tag{2}$$

where the free part of the Euclidean Lagrangian density is given by

$$\mathcal{L}_0(\varphi_1, \varphi_2) = \frac{1}{2}(\partial_\mu \varphi_1)^2 + \frac{1}{2} m_1^2 \varphi_1^2 + \frac{1}{2}(\partial_\mu \varphi_2)^2 + \frac{1}{2} m_2^2 \varphi_2^2, \tag{3}$$

and the interacting part is given by

$$\mathcal{L}_{int}(\varphi_1, \varphi_2) = V(\varphi_1) + \frac{\lambda_2}{2}(\varphi_1 \varphi_2)^2. \tag{4}$$

It is important to remark that the precise form of $V(\varphi_1)$ is not important for the construction of the effective action, but as we will see later in this section, the form of the $V(\varphi_1)$ is important to implement the decoupling theorem.

The action of the model is given by

$$S[\varphi_1, \varphi_2] = \int d^d x \, \mathcal{L}(\varphi_1(x), \varphi_2(x)), \tag{5}$$

and using Eq.(3) and Eq.(4) can be conveniently split up as

$$S[\varphi_1, \varphi_2] = S[\varphi_1(x)] + S_{\varphi_2}[\varphi_1(x), \varphi_2(x)], \tag{6}$$

$S[\varphi_1(x)]$ being the $\varphi_2(x)$-independent part of it. In order to obtain a derivative expansion of the effective action $\Gamma_{eff}[\varphi_1]$ we have to assume $m_2 >> m_1$. As usual, the operators $(-\Delta + m_1^2)^{-1}$ and

$(-\Delta+m_2^2)^{-1}$ must be used to define the free two-point Schwinger functions of the fields $\varphi_1(x)$ and $\varphi_2(x)$ respectively, and we must recall that Δ stands for the Laplacian operator in $\mathcal{R}^d$. According to the above, the free two-point Schwinger functions of both fields can be represented by

$$G(x-y;m_i) =< x|(-\Delta+m_i^2)^{-1}|y>,\ \ i=1,2 \tag{7}$$

and they obviously satisfy

$$(-\Delta+m_i^2)G(x-y;m_i) = \delta^d(x-y). \tag{8}$$

To obtain an effective action for the light modes of the theory, we integrate out the heavy field $\varphi_2(x)$ in the functional integral and define the effective action of the light modes $\Gamma_{eff}[\varphi_1]$ by

$$e^{-\Gamma_{eff}[\varphi_1]} = \int [d\varphi_2]\, e^{-S[\varphi_1,\varphi_2]}. \tag{9}$$

Using Eq.(6) it is possible to write Eq.(9) as

$$e^{-\Gamma_{eff}[\varphi_1]} = e^{-S[\varphi_1]} \int [d\varphi_2]\, e^{-S_{\varphi_2}[\varphi_1,\varphi_2]}. \tag{10}$$

The first step in our calculation is straightforward, since we have

$$S_{\varphi_2}[\varphi_1,\varphi_2] = \int d^dx\, (\varphi_2(-\Delta+m_2^2)\varphi_2 + \frac{\lambda_2}{2}(\varphi_1\varphi_2)^2), \tag{11}$$

and using Eq.(11) the functional integral appearing in Eq.(10) can be performed by means of Gaussian integrations, yielding

$$e^{-\Gamma_{eff}[\varphi_1]} = e^{-S[\varphi_1]}(det\, O)^{-\frac{1}{2}}, \tag{12}$$

where we have

$$O(x,y;m_2) =< x|O|y >= (-\Delta_x + m_2^2 + \lambda_2\varphi_1^2(x))\delta^d(x-y). \tag{13}$$

Consequently, the effective action for the light field $\varphi_1(x)$ is given by

$$\Gamma_{eff}[\varphi_1] = S[\varphi_1] + \frac{1}{2} tr\, ln\, O. \tag{14}$$

Dropping a term that contributes trivially to the effective action $\Gamma_{eff}[\varphi_1]$, we get

$$\Gamma_{eff}[\varphi_1] = S[\varphi_1] + \frac{1}{2} tr\, ln(1 + \lambda_2(-\Delta_x + m_2^2)^{-1}\varphi_1^2). \tag{15}$$

There are many ways to evaluate the Fredholm determinant, defined by the above equation. Using a series expansion, it is possible to rewrite Eq.(15) as

$$\Gamma_{eff}[\varphi_1] = S[\varphi_1] + \frac{1}{2}\sum_{k=1}^{\infty} \frac{(-1)^k}{k} tr(\lambda_2(-\Delta_x + m_2^2)^{-1}\varphi_1^2))^k, \tag{16}$$

or in a more compact notation

$$\Gamma_{eff}[\varphi_1] = S[\varphi_1] + \sum_{k=1}^{\infty} \Gamma^{(k)}[\varphi_1], \tag{17}$$

where each term of the series $\Gamma^{(k)}[\varphi_1]$ is given by Eq.(16). Let us study the first non trivial contribution of this series, namely, the term $k = 1$ which in fact corresponds to a one-loop diagram. It is explicitly given by

$$\Gamma^{(1)}[\varphi_1] = \frac{\lambda_2}{2} \int d^d x\, G(x-x; m_2)\, \varphi_1^2(x). \tag{18}$$

Using the Fourier representation for the two-point Schwinger function associated with the heavy field $\varphi_2(x)$, and defining a new coupling constant $\sigma = \lambda_2 \mu^{4-d}$, where μ is the usual dimensional parameter that appears in the dimensional regularization procedure, we readily obtain

$$\Gamma^{(1)}[\varphi_1] = \frac{\sigma}{2(2\sqrt{\pi})^d}\Gamma(1-\frac{d}{2})(m_2)^{d-2}\int d^dx\ \varphi_1^2(x). \tag{19}$$

Now, the Gamma function $\Gamma(z)$ is a meromorphic function of the complex variable z with simple poles at the points $z = 0, -1, -2, ..$ In the neighborhood of any of its poles $z = -n$, for $n = 0, 1, 2, ..$ $\Gamma(z)$ has a representation given by

$$\Gamma(z) = \frac{(-1)^n}{n!}\frac{1}{(z+n)} + \Omega(z+n), \tag{20}$$

where $\Omega(z+n)$ stands for the regular part of the analytic extension of $\Gamma(z)$. Note that for odd dimensions $\Gamma^{(1)}[\varphi_1]$ is completely regular while in even dimensions ($d = 2\ell\ \ell = 1, 2, ...$) there are singularities in the dimensional regularized quantity. By using the standard dimensional regularization prescription $d = 2\ell - \epsilon$ and since the Eq.(19) is quadratic in the field $\varphi_1(x)$, the divergence can be absorbed in the renormalized $\varphi_1(x)$ mass. Consequently, we define the renormalized mass for the light field as

$$m_{1R}^2 = m_1^2 + \frac{\sigma m_2^{d-2}}{(4\pi)^{\frac{d}{2}}}\left[\frac{(-1)^{\frac{d}{2}-1}}{(\frac{d}{2}-1)!}\frac{2}{\epsilon} + \Omega(\frac{\epsilon}{2})\right]. \tag{21}$$

For the odd dimensional case there are no poles, but we have the same situation. We have thus shown that at the one-loop approximation, the first correction to the effective action given by $\Gamma^{(1)}[\varphi]$ that we obtain integrating out the heavy field $\varphi_2(x)$ is just a modification of the value of the renormalized mass associated with the light field.

We will now show that the second correction to the effective action given by $\Gamma^{(2)}[\varphi_1]$ only modifies the value of the coupling constant of the field $\varphi_1(x)$. To this end, let us study the second term of the series in Eq.(17). It corresponds to a one-loop diagram and is actually given by

$$\Gamma^{(2)}[\varphi_1] = \frac{\sigma^2}{4}\int d^dx\int d^dy\, G(y-x;m_2)G(x-y;m_2)\varphi_1^2(x)\varphi_1^2(y), \tag{22}$$

which by upon substitution of the free two-point Schwinger function associated with the $\varphi_2(x)$ heavy field and the introduction of $I(p^2,m_2^2)$ as

$$I(p^2,m_2^2) = \frac{1}{(2\pi)^d}\int d^dq \frac{m_2^{4-d}}{(q^2+m_2^2)((p+q)^2+m_2^2)}, \tag{23}$$

can be written as

$$\Gamma^{(2)}[\varphi_1] = \frac{\sigma^2}{4(2\pi)^d}\int d^dx\int d^dy\,\varphi_1^2(x)\varphi_1^2(y)\int d^dp\, e^{-ip(y-x)}m_2^{d-4}I(p^2,m_2^2). \tag{24}$$

In the regularization and renormalization procedure we have to eliminate the poles and their residues adding counterterms in the Lagrangian density, consequently let us study the $I(p^2,m_2^2)$. Using the Feynman parametrization [29], it is possible to write $I(p^2,m_2^2)$ as

$$I(p^2,m_2^2) = N_d(-\frac{1}{\epsilon}+O(\epsilon))(1-\frac{d}{2})\int_0^1 dt(\frac{p^2}{m_2^2}t(1-t)+1)^{\frac{d}{2}-2}, \tag{25}$$

where N_d is the area of the sphere $S_{d-1}/(2\pi)^d$. The expression given by Eq.(25) contains a power of a binomial in a flat Euclidean d-dimensional space. When d is even, the power is an integer and the simple use of Newton's binomial theorem will give us a very direct way of evaluating $I(p^2,m_2^2)$. When d is odd, the expansion of $(1+\frac{p^2}{m_2^2}t(1-t))^{\frac{d}{2}-2}$ yields an infinite power series. Since we are using dimensional regularization we have an infinite power series. Note that the generalization of the binomial series is valid for any complex exponent p. In other words we have an everywhere convergent power series in p, hence a continuous function on p in the complex plane [30]. If we define

$$C_r^0 = 1, \quad C_r^1 = \frac{r}{1!}, \quad C_r^2 = \frac{r(r-1)}{2!} \ldots \tag{26}$$

until

$$C_r^k = \frac{r(r-1)..(r-k+1)}{k!}, \tag{27}$$

where $r = \frac{d}{2} - 2$, it is possible to write $I(p^2, m_2^2)$ as

$$I(p^2, m_2^2) = (1 - \frac{d}{2})N_d(-\frac{1}{\epsilon} + O(\epsilon)) \sum_{k=0}^{\infty} C^k_{\frac{d}{2}-2} \frac{p^{2k}}{m_2^{2k}} \int_0^1 dt(t(1-t))^k. \tag{28}$$

Let us use the definition of Euler's integral of first kind $B(\alpha, \beta)$ given by [31].

$$B(\alpha, \beta) = \int_0^1 dx\, x^{\alpha-1}(1-x)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \; Re\,\alpha > 0, Re\,\beta > 0. \tag{29}$$

Substituting Eq.(28) and Eq.(29) in Eq.(24) we find that the second term of the series that represents the effective action, $\Gamma^{(2)}[\varphi_1]$ can be written as

$$\begin{aligned}\Gamma^{(2)}[\varphi_1] \quad &= \frac{\sigma}{4(2\pi)^d} \int d^d x \int d^d y\, \varphi_1^2(x)\varphi_1^2(y) \int d^d q\; e^{-ip(y-x)} m_2^{d-4} \\ &((1 - \frac{d}{2})N_d(-\frac{1}{\epsilon} + O(\epsilon)) \sum_{k=0}^{\infty} C^k_{\frac{d}{2}-2} \frac{p^{2k}}{m_2^{2k}} B(k+1, k+1).\end{aligned} \tag{30}$$

If we choose the self-interacting part of the field $\varphi_1(x)$ to be $\lambda_1\varphi_1^4(x)$, it is possible to define the renormalized coupling constant λ_R subtracting the polar part. Consequently, the effective action for the $\varphi_1(x)$ field is given by

$$\Gamma_{eff}[\varphi_1] = \frac{1}{2}\int d^dx\ \varphi_1(x)(-\Delta+m_1^2)\varphi_1(x)+\lambda_R\int d^dx\ \varphi_1^4(x)$$
$$+\frac{\sigma^2}{4!(4\pi)^2 m_2^2}\int d^dx\ \varphi_1^2(x)(-\Delta+m_1^2)\varphi_1^2(x)+O(\frac{\Delta}{m_2^2})^2. \tag{31}$$

Note that the terms $k = 3, 4..$ are not divergent (in a four dimensional theory) and although they contribute to the effective action, in the limit $m_2 \to \infty$, the heavy field $\varphi_2(x)$ decouples from the light field $\varphi_1(x)$. The effect of the heavy field appears only modifying the values of the renormalized mass m_{1R} and the coupling constant λ_R. Thus we showed that the Euclidean version of the Appelquist-Carazzone decoupling theorem works in this specific model. Another well known example where the decoupling theorem can be used is in quantum electrodynamics, where for energies much lower than the electron mass it is possible to construct a derivative expansion of the Maxwell field effective action integrating out the Dirac field. This is an expected result, since we known that the theorem is valid for renormalizable theories without spontaneous symmmetry breaking or chiral fermions. The above discussion justifies the approach used by some authors that have been using the Euler-Heisenberg Lagrangian density to investigate the radiative correction to the Casimir effect [10], although these radiative corrections are of no phenomenological significance as was pointed out by Melnikov [14]. For a careful discussion of effective Lagrangians in quantum electrodynamics, see for example ref. [32].

It is important to stress that instead of obtain also a effective theory for the light field as have been discussed by many authors in finite temperature field theory [33], assuming some ultraviolet cut-off Λ and integrating over the Fourier modes with wavelenght smaller than Λ^{-1}, we are assuming that the functional integral must be taken over the space of the functions that vanish on the boundaries. One way to implement this is to introduce delta functions in the functional integral. This is equivalent to evaluate the functional integral over a space of functions that satisfy the boundary conditions. This is the procedure that we are addopting. It is clear that this procedure will introduce additional surface divergences that can be eliminated by surface counterterms, and in the end we have the effective model for the light modes that satisfies boundary conditions over some surfaces.

3 Finite size effects and the two and four-point Schwinger functions in the one-loop approximation

In the last section, we studied a very simple model of two Euclidean massive scalar fields where the decoupling theorem can be used after the construction of an effective theory of the light field. We have shown that in our model of two massive self-interacting scalar fields, the heavy modes associated with the $\varphi_2(x)$ field completely decouple from the light ones associated with the light field $\varphi_1(x)$ in the limit $m_2 \to \infty$. In the case of a $\lambda_1\varphi_1^4(x)$ self-coupling, the only effect of the former is a modification of the mass m_1, and the coupling constant of the light field. We are reducing the problem in this manner since we are able to concentrate in such a one field theory, i.e., we will consider a $\lambda_1\varphi^4(x)$ self-interacting model. We will consider that the field $\varphi(x)$ depends on $d-1$ unbounded coordinates that we call $\vec{r}$, and one bounded coordinate to which we will refer to as z that will be assumed to lie in the interval $[0, L]$. If we exclude the possibility of periodic or anti-periodic boundary conditions, this choice obviously breaks the full translational invariance because we have to assume boundary conditions on the hyperplanes $z = 0$ and $z = L$.

To write the full renormalized action for the theory with boundaries we need two regulators, the first one being the usual ϵ that is introduced in the dimensional regularization procedure and the second one that we call η representing the distance to a boundary. According to this the full renormalized action must be given by [24]:

$$\begin{aligned} S(\varphi) \quad &= \int_0^L dz \int d^{d-1}r \Big(\frac{A(\epsilon)}{2}(\partial_\mu\varphi)^2 + \frac{B(\epsilon)}{2}\varphi^2 + \frac{C(\epsilon)}{4!}\varphi^4\Big) \\ &+ \int d^{d-1}r \big(c_1(\eta)\varphi^2(\vec{r}, 0) + c_2(\eta)\varphi^2(\vec{r}, L)\big) \\ &+ \int d^{d-1}r \big(c_3(\eta)\varphi^4(\vec{r}, 0) + c_4(\eta)\varphi^4(\vec{r}, L), \end{aligned} \tag{32}$$

where $A(\epsilon)$, $B(\epsilon)$ and $C(\epsilon)$ are the usual coefficients for the bulk counterterms and the coefficients $c_i(\eta)$ $i = 1, ..4$, which depend on the boundary conditions for the field, are the coefficients for the surface counterterms. As usual all of these coefficients must be calculed order by order in perturbation theory. We are considering two different possibilities for the boundary conditions, namely Dirichlet-Dirichlet (DD) and Neumann-Neumann (NN) boundary conditions. These boundary conditions are given respectively by

$$\varphi(\vec{r}, z)|_{z=0} = \varphi(\vec{r}, z)|_{z=L} = 0, \tag{33}$$

and

$$\frac{\partial}{\partial z}\varphi(\vec{r}, z)|_{z=0} = \frac{\partial}{\partial z}\varphi(\vec{r}, z)|_{z=L} = 0. \tag{34}$$

The system we are interested is invariant only under translations along the direction parallel to the plates, implying that what is conserved is not the full momentum but the $(d-1)$ dimensional parallel momentum $\vec{p}$. For such conditions, a more convenient representation for the n-point Schwinger functions is a mixed $(\vec{p}, z)$ one. A Euclidean scalar field $\varphi(x)$ satisfying certain homogeneous boundary conditions on $z = 0$ and L can be expanded in Fourier series as:

$$\varphi(\vec{r}, z) = \frac{1}{(2\pi)^{\frac{d-1}{2}}} \sum_n u_n(z) \int d^{d-1}p\, \phi_n(\vec{p})\, e^{i\vec{p}.\vec{r}}, \tag{35}$$

where $\vec{p}$ is the continuum parallel momentum, and $u_n(z)$ stands for the eigenfunctions of the operator $-\frac{d^2}{dz^2}$:

$$-\frac{d}{dz^2} u_n(z) = k_n^2 u_n(z), \tag{36}$$

where $k_n = \frac{n\pi}{L}$, $n = 1, 2..$ for DD b.c and $n = 0, 1, 2..$ for NN boundary conditions respectively. The main difference between both boundary conditions being the presence of the zero mode. The free two-point Schwinger function for the theory can be expressed in the following form:

$$G_0^{(2)}(\vec{r}, z, \vec{r'}, z') = \frac{1}{(2\pi)^{d-1}} \sum_n u_n(z) u_n^*(z') \int d^{d-1}p \frac{e^{i\vec{p}.(\vec{r}-\vec{r'})}}{(\vec{p} + k_n^2 + m^2)}. \tag{37}$$

Note that in this section we have changed the notation as follows: $m_1 \to m$ and also $G(x, x'; m_1) \to G_0(x, x')$. It is useful to define also $\vec{\rho} = \vec{r} - \vec{r'}$. When considering DD boundary conditions, one finds that the free two-point Schwinger function is explicitly given by

$$G_0^{(2)}(\vec{\rho}, z, z') = \frac{2}{L} \sum_{n=1}^{\infty} \sin(\frac{n\pi z}{L}) \sin(\frac{n\pi z'}{L}) I_n(L, m, d, \vec{\rho}), \tag{38}$$

where

$$I_n(L,m,d,\vec{\rho}) = \frac{1}{(2\pi)^{d-1}} \int d^{d-1}p \frac{e^{i\vec{p}.\vec{\rho}}}{(\vec{p}^2 + (\frac{n\pi}{L})^2 + m^2)}. \tag{39}$$

It is clear that the family of $I_n(L,m,d,\vec{\rho})$ functions can be thought of as the free propagators of a tower of massive scalar fields in $(d-1)$ dimensions, the effective mass of each mode being given by $M_n^2 = m^2 + (\frac{n\pi}{L})^2$. This is to be expected since our theory has been formulated in a compactified space. From an even simpler point of view, $I_n(L,m,d,\vec{\rho})$ is nothing but the Fourier transform of a "spherically" $(SO(d-1))$ symmetric function of the parallel momentum $\vec{p}$.

We begin the study of the interacting theory by building the one-loop correction $(G_1^{(2)}(\lambda_1, x, x'))$ to the bare two-point Schwinger function $G_0^{(2)}(x,x')$, for both the DD and NN boundary conditions. Using the Feynman rules we have

$$G_1^{(2)}(\lambda_1, \vec{r}_1, z_1, \vec{r}_2, z_2) = \frac{\lambda_1}{2} \int d^{d-1}r \int_0^L dz\; G_0^{(2)}(\vec{r}_1 - \vec{r}, z_1, z) G_0^{(2)}(\vec{0}, z) G_0^{(2)}(\vec{r} - \vec{r}_2, z, z_2). \tag{40}$$

Here we would like to point out that even though the functions $G_0^{(2)}(\vec{r}_1 - \vec{r}_2, z_1, z_2)$ and $G_0^{(2)}(\vec{r}_2 - \vec{r}_3, z_2, z_3)$ are singular at coincident points ($\vec{r}_1 = \vec{r}_2$, $z_1 = z_2$) and ($\vec{r}_2 = \vec{r}_3$, $z_2 = z_3$), the singularities are integrable for points outside the plates. Using the notation $G_0^{(2)}(\vec{0}, z) = T_{DD}(L,m,d,z)$, a straightforward substitution yields the order λ_1 correction to the bare two-point Schwinger function in the one-loop approximation, for the case of Dirichlet boundary conditions:

$$\begin{aligned} G_1^{(2)}(\lambda_1, \vec{r}_1 - \vec{r}_2, z_1, z_2) \quad &= \frac{2\lambda_1}{(2\pi)^{d-1}L^2} \int_0^L dz \sum_{n,n'=1}^{\infty} \sin(\frac{n\pi z_1}{L}) \sin(\frac{n\pi z}{L}) \sin(\frac{n'\pi z}{L}) \sin(\frac{n'\pi z_2}{L}) \\ &\int d^{d-1}p \frac{e^{i\vec{p}(\vec{r}_1 - \vec{r}_2)}}{(\vec{p}^2 + (\frac{n\pi}{L})^2 + m^2)(\vec{p}^2 + (\frac{n'\pi}{L})^2 + m^2)} T_{DD}(L,m,d,z), \end{aligned} \tag{41}$$

where, since we are using dimensional regularization techniques, we have introduced a dimensional parameter μ, defining a dimensionless coupling constant $\lambda = \lambda_1 \mu^{4-d}$, and the expression for the amputated one-loop two-point function $T_{DD}(L,m,d,z)$ is given by

$$T_{DD}(L,m,d,z) = \frac{2}{(2\pi)^{d-1}L} \sum_{n=1}^{\infty} \sin^2(\frac{n\pi z}{L}) \int d^{d-1}p \frac{1}{(\vec{p}^2 + (\frac{n\pi}{L})^2 + m^2)}. \tag{42}$$

In the case of Neumann-Neumann boundary conditions the expression for the amputated one-loop two-point function can also be found following the same procedure, and it is given by

$$\begin{aligned} T_{NN}(L,m,d,z) \quad &= \frac{1}{(2\pi)^{d-1}L} \int d^{d-1}k \frac{1}{(\vec{k}^2 + m^2)} \\ &+ \frac{2}{(2\pi)^{d-1}L} \sum_{n=1}^{\infty} \cos^2(\frac{n\pi z}{L}) \int d^{d-1}p \frac{1}{(\vec{p}^2 + (\frac{n\pi}{L})^2 + m^2)}. \end{aligned} \tag{43}$$

Both $T_{DD}(L,m,d,z)$ and $T_{NN}(L,m,d,z)$ diverge in their continuum momenta integrals and also in the discrete mode summation. Using the Feynman rules, $G_2^{(4)}(\lambda, x_1, x_2, x_3, x_4)$, i.e., the $O(\lambda^2)$ correction to the bare one-loop four-point Schwinger functions, is given by

$$\begin{aligned} G_2^{(4)}(\lambda, \vec{r}_1, z_1, \vec{r}_2, z_2, \vec{r}_3, z_3, \vec{r}_4, z_4) \quad &= \quad \frac{\lambda^2}{2} \int d^{d-1}r \int d^{d-1}r' \int_0^L dz \int_0^L dz' \; G_0^{(2)}(\vec{r}_1 - \vec{r}, z_1, z) \\ &G_0^{(2)}(\vec{r}_2 - \vec{r}, z_2, z)(G_0^{(2)}(\vec{r} - \vec{r'}, z, z'))^2 \\ &G_0^{(2)}(\vec{r'} - \vec{r}_3, z', z_3) G_0^{(2)}(\vec{r'} - \vec{r}_4, z', z_4). \end{aligned} \tag{44}$$

Again, all G_0's are singular at coincident points, but the singularities are integrable for points outside the plates, except for $G_0^{(2)}(\vec{r} - \vec{r'}, z, z')$.

In the next section we will begin the renormalization program for the massless one-loop two-point Schwinger functions for the case of Dirichlet-Dirichlet boundary condition. The study of the complementary set of boundary conditions, namely NN boundary conditions can be performed along the same lines. When the fields satisfy NN boundary conditions that infrared divergences for massless fields appear, and in fact, such divergences come from the zero mode contribution, so the two-point Schwinger function for the case of Dirichlet-Dirichlet boundary conditions is IR finite for $m = 0$. For the case of NN we must have a finite Euclidean volume to regularize the theory in the infrared. Another way to deal with the infrared divergences in the case of Neumann-Neumann boundary condition in to perform a resummation of the daisy diagrams [34] [35] [36] [37]. Although this procedure is standard in the study of scalar models where the translational invariance is maintained, for systems where the translational invariance is broken, the problem of how to carry out the resummation program still remains open.

4 The regularized one-loop two and four-point Schwinger functions

In this section we would like to discuss in detail how to implement the one-loop renormalization program in finite size systems where flat surfaces break the translational invariance. Thus, the aim of this section is first to analyze the structure of the divergences associated with the one-loop two and also four-point function for the case of Dirichlet-Dirichlet boundary boundary conditions.

The amputated one-loop two-point Schwinger function $T_{DD}(L, m, d, z)$ can be decomposed in a translational invariance part and another one that breaks the translational invariance, indeed using algebraic identities [31] [38] one gets

$$T_{DD}(L, m, d, z) = f_1(L, m, d) - f_2(L, m, d, z), \tag{45}$$

where the functions $f_1(L, m, d)$ and $f_2(L, m, d, z)$ are given respectivelly by

$$f_1(L, m, d) = \frac{1}{2(2\pi)^{d-1}L} \sum_{n=-\infty}^{\infty} \int d^{d-1}p \frac{1}{(\vec{p}^2 + (\frac{n\pi}{L})^2 + m^2)} \tag{46}$$

and

$$f_2(L, m, d, z) = \frac{1}{2(2\pi)^{d-1}} \int d^{d-1}p \frac{1}{\sqrt{\vec{p}^2 + m^2}} \frac{cosh((L - 2z)\sqrt{\vec{p}^2 + m^2})}{senh(L\sqrt{\vec{p}^2 + m^2})}. \tag{47}$$

The amputated one-loop two-point Schwinger function for the Neumann-Neumann boundary conditions, $T_{NN}(L, m, d, z)$ can be similarly split up as

$$T_{NN}(L, m, d, z) = f_1(L, m, d) + f_2(L, m, d, z) \tag{48}$$

The above decompositions of $T_{DD}(L, m, d, z)$ and $T_{NN}(L, m, d, z)$ have the same functional form and as we stated before, some of the divergences come purely from the bulk while others depend on the distance to the boundaries. Indeed, since $f_1(L, m, d)$ does not depend on z, it only carries information about the divergences on the bulk. These divergences occur not only in the discrete sums but also in the momentum integrations. After the identification: $\beta \equiv 2L$ $f_1(L, m, d)$ is formally proportional to the amputated one-loop two-point function of the theory assuming that the system is in thermal equilibrium with a reservoir at temperature β^{-1}. To deal with the divergences of $f_1(L, m, d)$, or equivalently, the one-loop two-point Schwinger functions at finite temperature we have to do frequency sums and $(d-1)$ dimensional integrals for the continuum momenta. One way to perform the integrals with Matsubara sums is to analytic extend away from the discrete complex energies down to real axis with the replacement of the energy sums by contour integrals [39] [40]. Another way is to use dimensional regularization and afterwards to analytically extend the modified Epstein zeta function which appears after dimensional regularization. Direct use of dimensional regularization identities and the analytic extension of the modified Epstein zeta function in the sum given by Eq.(46) which defines $f_1(L, m, d)$, give us a polar part (size independent) plus a size-dependent analytic correction. The mass counterterm (the principal part of the Laurent series of the analytic regularized quantity) generated by $f_1(L, m, d)$ is size independent, because the finite temperature field theory has no temperature dependent counterterm. Observe that the non translational invariant part of the amputated one-loop two-point Schwinger function expressed by $T_{DD}(L, m, d, z)$ and $T_{NN}(L, m, d, z)$ have the same z dependent part in modulus but with opposite signs.

Since we have shown that the $T_{DD}(L, m, d, z)$ and $T_{NN}(L, m, d, z)$ can be split into two functions $f_1(L, m, d)$ and $f_2(L, m, d, z)$ and since as we have just discussed the behavior of $f_1(L, m, d)$, we can now turn our attention to the study of the divergences contained in $f_2(L, m, d, z)$. We begin by an angular integration ($d^{d-1}p = p^{d-2}dp\, d\Omega_{d-1}$ and, $\int d\Omega_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})}$) that leads to an alternate expression for the non translational invariant part $f_2(L, m, d, z)$, namely

$$f_2(L, m, d, z) = \frac{1}{2}h(d)\int_0^\infty dp \frac{p^{d-2}}{\sqrt{p^2+m^2}} \frac{cosh((L-2z)\sqrt{p^2+m^2})}{sinh(L\sqrt{p^2+m^2})}. \tag{49}$$

Using the change of variables $s = \sqrt{p^2+m^2}$ in the above expression yields the following formula for $f_2(L,m,d,z)$:

$$f_2(L,m,d,z) = \frac{1}{2}h(d)\int_m^\infty ds(s^2-m^2)^{\frac{d-3}{2}} cosh((L-2z)s)(\sinh Ls)^{-1}, \tag{50}$$

where $h(d)$ is an analytic function of d given by $h(d) = \frac{1}{2(2\sqrt{\pi})^{d-1}}\frac{1}{\Gamma(\frac{d-1}{2})}$. We now start studying the massless case following Fosco and Svaiter [41]. In fact, we are particularly interested in examining the limits ($z \to 0^+$ and $z \to L^-$) which do obviously contain the information about the effects of the boundaries. In order to fulfill this goal we introduce two new variables $x = Ls$ and $q = zs$ in terms of which we can write $f_2(L,m,d,z)|_{m=0}$ as

$$\begin{aligned} f_2(L,m,d,z)|_{m=0} &= \frac{h(d)}{2L^{d-2}}\int_0^\infty dx\, x^{d-3}(\coth x - 1)\cosh(\frac{2zx}{L}) \\ &+ \frac{h(d)}{2z^{d-2}}\int_0^\infty dq\, q^{d-3}e^{-2q}. \end{aligned} \tag{51}$$

The second term of Eq.(51) give us the well known result that for a massless scalar field in $d = 4$ the one-loop vacuum fluctuations diverges as $\frac{1}{z^2}$ if we approach the boundary at $z = 0$ [42]. The other term of Eq.(51) should behave as $\frac{1}{(L-z)^{d-2}}$. To see this let us investigate the behavior of the first integral of $f_2(L,m,d,z)|_{m=0}$ near the boundary at $z = L$. In order to do this, we make use of two formulas involving the definition for the Gamma function, and also another well known integral representation for the product of the Gamma function times the Hurwitz zeta function given by

$$\int_0^\infty dx\, x^{\mu-1}e^{-\beta x}(\coth x - 1) = 2^{1-\mu}\Gamma(\mu)\zeta(\mu, \frac{\beta}{2}+1) \quad Re(\beta) > 0, \quad Re(\mu) > 1, \tag{52}$$

where $\zeta(z,a)$ is the Hurwitz zeta function defined by [31]

$$\zeta(z,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^z}, \quad Re(z) > 1, \quad a \neq 0, -1, -2... \tag{53}$$

From the definition of the Gamma function and Eq.(52) in Eq.(51) we may write the following closed expression

$$\begin{aligned} f_2(L,m,d,z)|_{m=0} &= \frac{h(d)}{2L^{d-2}} \left[2^{2-d}\Gamma(d-2) \left(\zeta(d-2, \frac{z}{L}+1) + \zeta(d-2, -\frac{z}{L}+1) \right) \right] \\ &+ \frac{1}{(2z)^{d-2}} h(d)\Gamma(d-2). \end{aligned} \tag{54}$$

From this last expression and using the definition of the Hurwitz zeta function giving by Eq.(53) it is evident that the regularized $f_2(L,m,d,z)|_{m=0}$ has two poles of order $(d-2)$, one at $z=0$ and another at $z=L$.

To study the massive case, from the expression given by Eq.(50) it is possible to write $f_2(L,m,d,z)$ in a more convenient way by:

$$f_2(L,m,d,z) = f_{21}(L,m,d,z) + f_{22}(L,m,d,z). \tag{55}$$

where $f_{21}(L,m,d,z)$ and $f_{22}(L,m,d,z)$ are

$$f_{21}(L,m,d,z) = \frac{1}{2} h(d) \int_m^{\infty} ds (s^2 - m^2)^{\frac{d-3}{2}} e^{-2zs}, \tag{56}$$

and

$$f_{22}(L,m,d,z) = \frac{1}{2} h(d) \int_m^{\infty} ds (s^2 - m^2)^{\frac{d-3}{2}} (\coth Ls - 1) \cosh 2zs. \tag{57}$$

Using an integral representation of the Bessel function of third kind or Macdonald's functions it is possible to find the following closed expression of $f_{12}(L,m,d,z)$ given by

$$f_{21}(L,m,d,z)=\frac{1}{2}\frac{1}{(2\sqrt{\pi})^{d-1}}(\frac{m}{z})^{\frac{d-2}{2}}K_{\frac{d-2}{2}}(2mz). \tag{58}$$

For small z and finite m we have the asymptotic formula $K_\nu(z)\approx 2^{\nu-1}\Gamma(\nu)z^{-\nu}$, thus for $z\to 0^+$, the function $f_{21}(L,m,d,z)$ diverges as $\frac{1}{z^{d-2}}$. To calculate $f_{22}(L,m,d,z)$ we will use the same method that we used in section II. Again note that it contains a power of a binomial. Making use of the generalized binomial formula gives

$$(1-\frac{m^2}{s^2})^{\frac{d-3}{2}}=\sum_{k=0}^{\infty}(-1)^k C^k_{\frac{d-3}{2}}(\frac{m}{s})^{2k}, \tag{59}$$

and introducing a new variable $u=Ls$ we obtain

$$f_{22}(L,m,d,z)=\frac{h(d)}{2L^{d-2}}\sum_{k=0}^{\infty}(-1)^k C^k_{\frac{d-3}{2}}(Lm)^{2k}\int_{Lm}^{\infty}du\,u^{d-3-2k}(\coth u-1)\cosh(\frac{2zu}{L}) \tag{60}$$

Our next step is to show that this result can be expressed in terms of the Hurwitz zeta function. A natural way to achieve the proof is to split $f_{22}(L,m,z,d)$ as a sum of two terms

$$f_{22}(L,m,d,z)=f_{22}^{<}(L,m,z,d)+f_{22}^{>}(L,m,z,d), \tag{61}$$

where

$$f_{22}^{<}(L,m,z,d)=-\frac{1}{4L^{d-2}}\sum_{k=0}^{k<\frac{d-3}{2}}C^{(1)}(d,k)(Lm)^{2k}\int_{Lm}^{\infty}du\,u^{d-3-2k}(\coth u-1)\cosh(\frac{2uz}{L}), \tag{62}$$

and

$$f_{22}^{>}(L,m,z,d)=-\frac{1}{4L^{d-2}}\sum_{k\geq\frac{d-3}{2}}^{\infty}C^{(1)}(d,k)(Lm)^{2k}\int_{Lm}^{\infty}du\,u^{d-3-2k}(\coth u-1)\cosh(\frac{2uz}{L}). \tag{63}$$

Here we have introduced $C^{(1)}(d,k) = (-1)^k C^k_{\frac{d-3}{2}} h(d)$ and also $C^{(2)}(d,k) \equiv \frac{\Gamma(d-2-2k)}{2^{d-3-2k}} C^{(1)}(d,k)$ it is possible to write Eq.(62) in the following way:

$$
\begin{aligned}
f^{<}_{22}(L,m,z,d) \;\; &= \;\; -\frac{1}{4L^{d-2}} \sum_{k=0}^{k<\frac{d-3}{2}} C^{(2)}(d,k)(Lm)^{2k} \left(\zeta(d-2-2k, -\frac{z}{L}+1) + \zeta(d-2-2k, \frac{z}{L}+1) \right) \\
&\quad + \frac{1}{4L^{d-2}} \sum_{k=0}^{k<\frac{d-3}{2}} C^{(1)}(d,k)(Lm)^{2k} \int_0^{Lm} du\, u^{d-3-2k} (\coth u - 1) \cosh(\frac{2uz}{L}), \qquad (64)
\end{aligned}
$$

where the singularities of $f^{<}_{22}(L,m,z,d)$ appear at $z \to L$. Turning our attention to $f^{>}_{22}(L,m,z,d)$, it is clear that in the expression above we see that the surface divergences are the same as we studied before in the massless case.

We now turn our attention back to the four-point Schwinger function in the one-loop approximation. Introducing new variables as $u_{\pm} \equiv z \pm z'$, the two-point Schwinger function in the tree-level can be split into

$$
G_0^{(2)}(\vec{\rho}, z, z') = G_{+}^{(2)}(\vec{\rho}, u_{+}) + G_{-}^{(2)}(\vec{\rho}, u_{-}), \qquad (65)
$$

where making use of the definition of $I_n(L,m,d,\vec{\rho})$ given by Eq.(39) we have

$$
G_{\pm}^{(2)}(\vec{\rho}, u_{\pm}) = \mp \frac{1}{L} \sum_{n=1}^{\infty} cos(\frac{n\pi u_{\pm}}{L}) I_n(L,m,d,\vec{\rho}). \qquad (66)
$$

Before continue, let us present a explicit formula of the free two-point Schwinger function in terms of Bessel functions. Defining an analytic function $g(d)$ by

$$
g(d) = \frac{1}{\sqrt{\pi}(2\pi)^{\frac{d-1}{2}}} \frac{\Gamma(\frac{d-2}{2})}{\Gamma(\frac{d-3}{2})}, \qquad (67)
$$

it is possible to show that we can write $G^{(2)}_{\pm}(\rho, u_{\pm})$ as

$$G^{(2)}_{\pm}(\rho, u_{\pm}) = \mp\frac{g(d)}{\rho^{\frac{d-3}{2}}L}\sum_{n=1}^{\infty} cos(\frac{n\pi u_{\pm}}{L})((\frac{n\pi}{L})^2 + m^2)^{\frac{d-3}{4}} K_{\frac{d-3}{2}}(\rho(m^2 + (\frac{n\pi}{L})^2)^{\frac{1}{2}}) \tag{68}$$

Using Eq.(65) and the above formula gives us the explicit expression for the two-point Schwinger function in a generic d dimensional Euclidean space confined between two flat paralel hyperplanes where we assume Dirichlet-Dirichlet boundary conditions. It is hard to use the above expressions for $G^{(2)}_{\pm}(\rho, u_{\pm})$ to investigate the analytic structure of the four point function $G^{(4)}_{2}(\lambda, \vec{r}_1, z_1, \vec{r}_2, z_2, \vec{r}_3, z_3, \vec{r}_4, z_4)$, given by Eq.(44), for both the bulk and near the boundaries, Nevertheless from Eqs.(38) and Eq.(39) it is clear that the divergences of the four-point function in the one-loop approximation appear at coincident points and therefore the singular behavior is encoded in the polar part of $M(\lambda, L, m, d)$ given by

$$M(\lambda, L, m, d) = \lambda^2 \int d^{d-1}r \int d^{d-1}r' \int_0^L dz \int_0^L dz' F(\vec{r}, \vec{r}', z, z')(G^{(2)}_0(\vec{r} - \vec{r}', z, z'))^2, \tag{69}$$

where $F(\vec{r}, \vec{r}', z, z')$ is a regular function. As with the one-loop two point function, it is not difficult to realize that the above equation has two kinds of singularities, those comming from the bulk and those arising from the behavior near the surface. As before the behavior in the bulk is as that found in thermal field theory and consequently we will only discuss the singularities that arise from the boundaries. This can be done by studying the polar part of $\tilde{M}(\lambda, L, m, d)$ given by

$$\tilde{M}(\lambda, L, m, d) = \frac{\lambda^2}{2} \int_0^L dz \int_0^L dz' \mathcal{F}(z, z')(G^{(2)}_0(\vec{0}, z, z'))^2, \tag{70}$$

where $\mathcal{F}(z,z')$ is a regular function. Now, we recall that the form of $G^{(2)}_{\pm}(\rho,u_{\pm})|_{\rho=0}$ is given by,

$$G^{(2)}_{\pm}(\rho,u_{\pm})|_{\rho=0} = \mp\frac{1}{(2\pi)^{d-1}L}\sum_{n=1}^{\infty} cos(\frac{n\pi u_{\pm}}{L})\int d^{d-1}p\frac{1}{(\vec{p}^2+m^2+(\frac{n\pi}{L})^2)}, \tag{71}$$

from where it is not difficult to show that the free correlation function is given by

$$G^{(2)}_{0}(\rho,z,z')|_{\rho=0} = f_2(L,m,d,\frac{u_-}{2}) - f_2(L,m,d,\frac{u_+}{2}). \tag{72}$$

For the sake of simplicity we will discuss only the massless case since the singularities of the massive case have the same structure as the massless one. The function $f_2(L,m,d,\frac{u_+}{2})$ is non singular in the bulk, i.e. in the interior of the interval $[0,L]$, while $f_2(L,m,d,\frac{u_-}{2})$ has a singularity along the line $z=z'$. Indeed, closer inspection shows that for $0\leq z,z'\leq L$ the only singularities are those at $u_+=0$, $u_+=2L$ and also $u_-=0$. The former two are genuinely boundary singularities (the two conditions imply $z,z'\to 0$ or $z,z'\to L$) while the other comming from $z=z'$ in the whole domain is just the standard bulk singularity. In fact, using the structure of the two point function and showing just those terms from which singularities might arise, one finds that the counterterms for $\tilde{M}$ are given by

$$-\text{pole}\int_0^L dz\int_0^L dz'[\frac{C_1}{(z+z')^{d-2}}+\frac{C_2}{(2L-z-z')^{d-2}}+\frac{C_3}{(z-z')^{d-2}}+...]^2. \tag{73}$$

where $C_i, i=1,..3$ are regular functions that do not depend on z or z'. From this discussion it is clear that in order to render the field theory finite, we must introduce surface terms in the action. This is a general statement. For any fields that satisfy boundary condition that breaks the translational invariance, in addition to the usual bulk counterterms, it is sufficient to introduce surface counterterms in the action to render the theory finite.

5 Boundary effects and renormalization

In the last section we present the one-loop renormalization of the $\lambda\varphi^4$ model, and we considered that the field $\varphi(x)$ depends on $d-1$ unbounded coordinates that we call $\vec{r}$ and one bounded coordinate defined in the interval $[0, L]$. The boundary conditions on the hyperplanes $z = 0$ and $z = L$ are the usual Dirichlet-Dirichlet and also Neumann-Neumann boundary conditions.

In this section we would like to discuss briefly the global approach, used to define the Casimir energy associated with any field in the presence of surfaces with well defined geometric shape. The crucial conceptual question is the meaning of the renormalized vacuum energy associated with any field in the presence of any macroscopic structure that divides the space into the internal and the external region. It is important to keep separate different situations. In the case of the parallel plates, the region outside the plates is the union of two simple connected domains and both have the same geometry of the internal region. In this situation the Casimir renormalization procedure is well defined and the renormalized vacuum energy is unambiguous defined. In the case of the spherical an the cylindrical shell, the contribution of the exterior modes are not cancelled out in the Casimir renormalization procedure. It is not difficult to understand the origin of the problem, as has been extensively discussed in the literature. If we are assuming perfectly reflecting boundaries, by the Weyl theorem we know that the assymptoptic distribution of eigenvalues of some elliptic differential operator is related with the geometric invariants associated with the surface where the field satisfy some boundary condition [43] [44]. Consequently in the regularized energy we have divergent terms proportional to the volume, area, etc. It the Casimir definition of the renormalized vacuum energy it is not possible to cancel the area contribution for a generic surface. The generalization of the Weyl's expansion can be done investigating the trace of the heat kernel on a specified manifold with boundary. We conclude that the assumption of perfect conducting static boundaries with a generic shape introduces new problems in order to define the renormalized vacuum energy of a quantum system in the presence of these macroscopic objects [45] [46] [47] [48]. If someone insists in the assumption of perfect conducting boundaries there are different ways to solve the problem of infinite energy associated with the configuration. One is to introduce counterterms concentrated on the boundaries, as has been discussed by the authors that use the generalized zeta function method [49]. A different approach is to smooth out the plate surface by a classical potential [50] [51]. It is clear that the introduction of a classical potential $V(x)$ does not solve the problem of surface counterterms since in this situation we have to renormalize the potential. A very simple situation is the case of a background field where to compute the effective action we have to evaluate the the following Fredholm determinant where we are assuming that the positive potential is a large quantity.

$$D(V) = det\,(-\Delta + m^2 + V(x))(-\Delta + m^2)^{-1}. \tag{74}$$

For sufficient regular but large $V(x)$ it is possible to show that for $d = 4$ a counterterm quadratic in V is required in order to eliminate the divergences of the model [52]. Thus the introduction of a classical potential $V(x)$ trying to improve the unphysical boundary condition does not solve the problem of surface counterterms since in this situation we have to renormalize the potential. Instead of smoothing the plates surfaces, a more fruitful approach to avoid surface divergences, discussed by Kennedy et al [49] is to treat the boundary as a quantum mechanical object. This approach was developed recently by Ford and Svaiter [53] to produce finite values for the renormalized $< \varphi^2(x) >$ and other quantities that diverge as one approaches the classical boundary. We would like to stress that there will not be any surface divergences in a more exact treatment, however one can still make the case that surface counterterms are a useful phenomenological approach for dealing with the apparent surface divergences withouth going into the complexity of the more exact approach.

6 Conclusions

In this paper we discussed the approach of effective theory to perform calculations in field theory in the presence of macroscopic structures. We first assumed the theory of two interacting massive scalar fields $\varphi_1(x)$ and $\varphi_2(x)$ with masses m_1 and m_2 satisfying the condition $m_2 >> m_1$. Integrating out the modes of the field $\varphi_2(x)$ we obtained an effective Lagrangian density for $\varphi_1(x)$. In the limit $m_2 \to \infty$ the field $\varphi_2(x)$ decouples from $\varphi_1(x)$, the only effect of $\varphi_2(x)$ being modifying both the value of the renormalized mass m_1 and the coupling constant of the light field φ_1. Thus we considered the $\frac{\lambda}{4!}\varphi_1^4$ model on a d-dimensional Euclidean space, where all but one of the coordinates are unbounded. Translation invariance along the bounded coordinate, z, which lies in the interval $[0, L]$, is broken because of the boundary conditions (BC's) chosen for the hyperplanes $z = 0$ and $z = L$. Two different possibilities for these BC's boundary conditions are considered: DD and NN, where D denotes Dirichlet and N Neumann, respectively. The renormalization procedure up to one-loop order was implemented. The main result of our investigations is that in the presence of boundaries where the field satisfies some boundary condition, the augmented action with surface counterterms can deal with the surface divergences that appear in the one-loop Feynman diagrams.

There are several directions for future research in field theory in the presence of surfaces, of which we would like to emphasize two. The first one is to implement the renormalization program beyond the one-loop approximation, where overlapping divergences emerge. The second one is related to the infrared divergences. As we discussed, one way to deal with the infrared divergences in the case of Neumann-Neumann boundary condition in to perform a resummation of the daisy diagrams, although this procedure is standard in the study of scalar models at finite temperature, for systems where the translational invariance is broken, it is a open problem how to perform the resummation program. Both subjects are under investigation by the authors.

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